CONSTRUCTION OF SIX SIGMA BASED CONTROL CHART FOR MEAN USING STANDARD DEVIATION UNDER MODERATE DISTRIBUTION

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Abstract

Control chart can help in determining the source of errors by identifying the special and common causes of variations. A control chart can easily collect, organize and store information, calculate answers and present results in easy to understand graphs. It helps to record data and allows seeing when an unusual event, e.g., a very high or low observation compared with "typical" process performance, occurs, Samip Shah et. al (2010). Primarily, three types of control charts are employed to observe the disturbances in the process parameters. The large shifts are detected efficiently by Shewhart's (1931) control chart, whereas for small and medium instabilities, cumulative sum and exponentially weighted moving average control charts are used. Now the companies in developed and developing countries started applying Six Sigma initiatives in their manufacturing process, which results in lesser number of defects. The companies practicing six sigma initiatives are expected to produce 3.4 or less number of defects per million opportunities, a concept suggested by Motorola. In this paper an attempt is made to construct a six sigma based control chart for mean using standard deviation under Moderate distribution specially designed for the companies applying six sigma initiatives in their organization.

Keywords: Control Chart, Moderate distribution, Process control and Six Sigma.

1. Introduction

Statistical process control charts such as the Shewhart (1931) control chart, the cumulative sum control chart, Page (1961), and the exponentially weighted moving average control chart, Roberts (1959), are used to monitor product quality and detect special events that may be indicators of out-of-control situations. Such charts are designed on the assumption that a process being monitored will produce measurements that can be modelled with an independent and identically distributed normal distribution, when only the inherent sources of variability are present in the system, Thaga (2009). The companies, which are practicing Six Sigma, are expected to produce 3.4 or less number of defects per million opportunities. Radhakrishnan and Sivakumaran (2008) used the concept of six sigma in the construction of sampling plans such as single, double and repetitive group sampling plans indexed through Six Sigma Quality Levels (SSQLs) with Poisson distribution as the base line distribution. Radhakrishnan (2009) suggested single sampling plan indexed through Six Sigma quality levels (SSQLs) based on Intervened Random Effect Poisson Distribution and Weighted Poisson Distribution as the base line distributions. Radhakrishnan and Balamurugan (2016) constructed control chart based on six sigma initiatives for Cumulative – Sum. The control charts originated by W.A. Shewhart (1931) was based on 3 sigma control limits. If the same charts are used for the products of the companies which adopt six sigma initiatives in the process, then no point will fall outside the control limits because of the improvement in the quality. So a separate control chart is required to monitor the outcomes of the companies, which adopt six sigma initiatives. In this research article an attempt is made to construct a six sigma based control chart for mean using standard deviation under Moderate distribution.

2. Assumptions for the study

- Production managers involved in the study will be willing and able to learn the principles of evaluating control charts.
- Production managers involved in the study will have adequate knowledge and experience to make adjustments to an activity to improve the productivity of a process based on the data conveyed in the control charts.
- The activities to be studied will feature crews comprised of the same laborers and operators during the pre-intervention and intervention periods.

3. Conditions for application

- Six sigma based control limits will be used if the distribution is found to be Moderate.
- Companies adopt the concept of Six sigma based control limits using process capability under Moderate distribution in its processes.

4. Moderate distribution

Desai (2011) has developed moderate distribution, an alternative of normal distribution. In this distribution mean μ and mean deviation δ are parameters. Suppose the probability distribution function of a distribution of a random variable X is defined as,

$$f(x)=\frac{1}{\pi\delta}e^{-\frac{1}{\pi}\left(\frac{X-\mu}{\delta}\right)^2},\ -\infty<X<\infty,\ \delta>0$$

Suppose, X~ M (μ, δ), then the variable Z is defined as,

$$Z=\frac{X-\mu}{\delta},$$

has the probability distribution function defined as,

$$g(Z)=\frac{1}{\pi}e^{-\frac{1}{\pi}Z^2},\ -\infty<Z<\infty,$$

And this variable Z may be called standard moderate variate. Also, its distribution may be called standard moderate distribution.

5. Construction of six sigma based control chart for mean using standard deviation under Moderate distribution

Fix the tolerance level (TL) and process capability (C_P) to determine the process standard deviation ($\sigma_{MD:6\sigma}$). Apply the value of $\sigma_{MD:6\sigma}$ in the control limits $\bar{\bar{X}} \pm A_{MD:6\sigma}\sigma_{MD:6\sigma}$, to get the six sigma based control limits for mean using standard deviation under Moderate distribution. The value of $A_{MD:6\sigma}$ is obtained using $p(z \le z_{6\sigma}) = 1-\alpha_1, \alpha_1 = 3.4 \text{ x } 10^{-6}$ and z is a standard moderate variate. For a specified TL and C_P of the process, the value of σ (termed as $\sigma_{MD:6\sigma}$) is calculated from $c_p = \frac{TL}{6\sigma}$ using a JAVA Script for various combinations of TL and C_P.

The six sigma based control limits for mean using standard deviation under Moderate distribution are

$$\mathrm{UCL}_{MD:6\sigma} = \bar{\bar{X}} + \left(\frac{A_{MD:6\sigma}}{\sqrt{n}} \sigma_{MD:6\sigma} \right)$$

$$\mathrm{CL}_{MD:6\sigma} = \bar{\bar{X}}$$

$$\mathrm{LCL}_{MD:6\sigma} = \bar{\bar{X}} - \left(\frac{A_{MD:6\sigma}}{\sqrt{n}} \sigma_{MD:6\sigma} \right)$$

where $\sigma_{MD:6\sigma}$ is in place of $\frac{\bar{S}}{c_4}$ in 3 – Sigma control limits

6. Example

The example provided by Amitava Mitra (2001, Page No. 284) is considered here. The thickness of the magnetic coating on audio tapes is an important characteristic. Random samples of size 4 are selected, and the thickness is measured (microns) using an optical instrument. Table 3.3.1 shows the Mean $\bar{X}$ and standard deviation s for 20 samples. The specifications are 38 ± 4.5 microns. If a coating thickness is less than the specifications call for, that tape cane be used for a different purpose by running it through another coating operation.

Table 1: Mean ($\bar{X}_i$) and Standard deviation (S_i) of thickness of the magnetic coating on audio tapes

Sample	Sample Mean $\bar{X}$	Sample standard deviation, s
1	36.4	4.6
2	35.8	3.7
3	37.3	5.2
4	33.9	4.3
5	37.8	4.4
6	36.1	3.9
7	38.6	5.0
8	39.4	6.1
9	34.4	4.1
10	39.5	5.8
11	36.7	5.3
12	35.2	3.5
13	38.8	4.7
14	39.0	5.6
15	35.5	5.0
16	37.1	4.1
17	38.3	5.6
18	39.2	4.8
19	36.8	4.7
20	37.7	5.4

$$\bar{S} = \frac{\sum_{i=1}^{N} S_i}{N} = \frac{95.80}{20} = 4.790$$

6.1 Construction of control limits (3σ) for Mean using Standard Deviation

The 3σ control limits suggested by W.A. Shewhart (1931) for Mean using Standard deviation are

$$\bar{\bar{X}} \pm A_3\bar{S}$$

$$UCL_{3\sigma} = \bar{\bar{X}} + A_3\bar{S} = 37.175 + (1.628 \times 4.790) = 45$$

$$\text{Central line } CL_{3\sigma} = \bar{\bar{X}} = 37.2$$

$$LCL_{3\sigma} = \bar{\bar{X}} - A_3\bar{S} = 37.175 - (1.628 \times 4.790) = 29.4$$

where

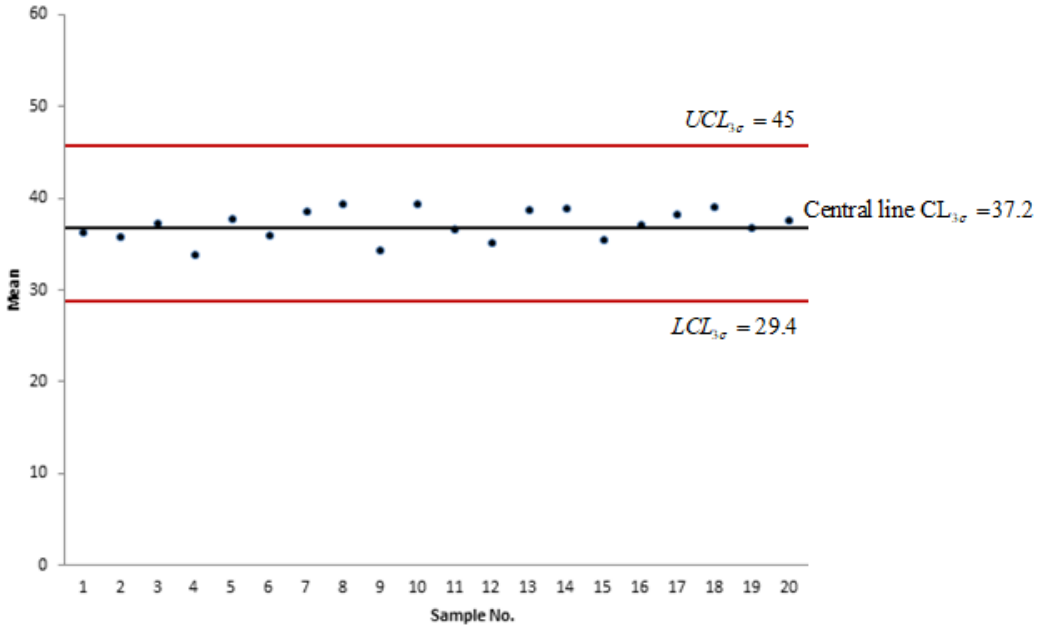

Figure 1: 3σ Shewhart control limits

From the resulting Figure 1, it is clear that the process is in control, since the entire sample numbers lie inside the control limits.

6.2 Six sigma based control limits for mean using standard deviation under Moderate distribution

For a given TL=2.6 and C_p=1.5, the value of $\sigma_{MD:6\sigma}$ can be obtained as 0.29 and the value of $z_{6\sigma}$ obtained from the Moderate distribution as 5.64 and hence, the value of $A_{MD:6\sigma}$ is obtained as 2.82.

The six sigma based mean using standard deviation chart under Moderate distribution for a specified TL, $A_{MD:6\sigma}$ and n is

$$UCL_{MD:6\sigma} = \bar{\bar{X}} + \left(\frac{A_{MD:6\sigma}}{\sqrt{n}}\sigma_{MD:6\sigma}\right) = 37.175 + (2.82 \times 0.29) = 37.9928$$

$$CL_{MD:6\sigma} = \bar{\bar{X}} = 37.175$$

$$LCL_{MD:6\sigma} = \bar{\bar{X}} - \left(\frac{A_{MD:6\sigma}}{\sqrt{n}}\sigma_{MD:6\sigma}\right) = 37.175 - (2.82 \times 0.29) = 36.3572$$

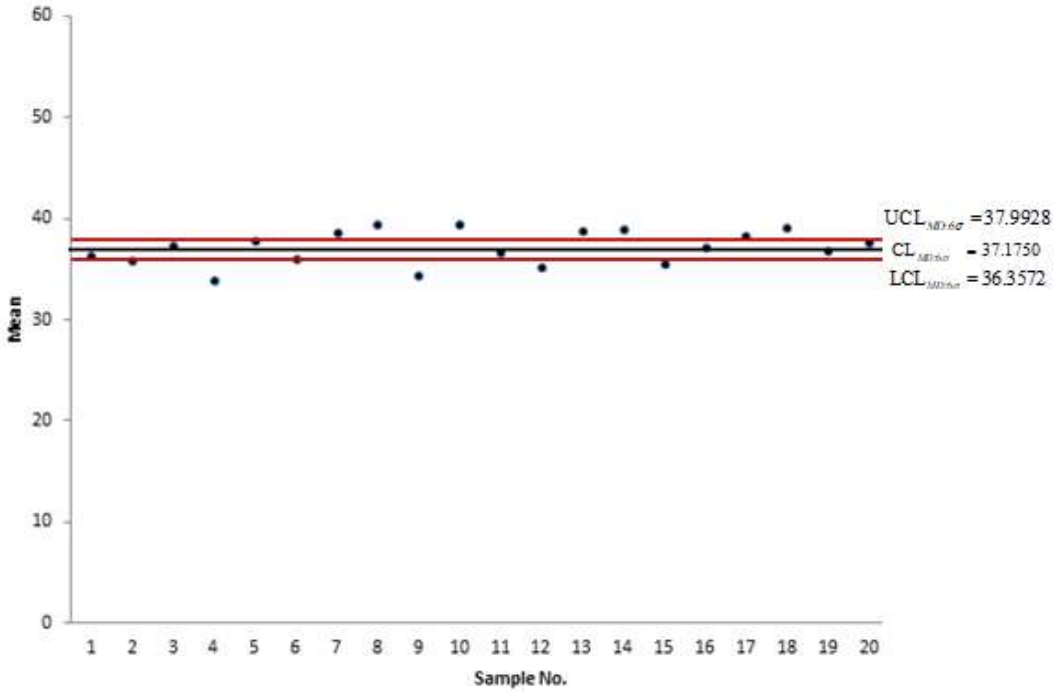

Figure 2: Six sigma based control limits for mean using standard deviation under MD

It is clear from the Figure 2, the process is out of control, since the sample points 7, 8, 10, 13, 14, 17 and 18 are lie outside the upper control limit and the sample points 2, 4, 9, 12 and 15 are lie outside the lower control limit. It is found from the Figures 1 and 2, the process is in Statistical control when 3 – Sigma control limits are used and the process is not in Statistical Control when Six Sigma based control limits under Moderate distribution are adopted. It is clear that the product/Service is not in good Quality as expected using Six Sigma Initiatives, so a correction is needed in the process/System.

7. Conclusion

In this paper, a procedure is given to construct a six sigma based control chart for mean using standard deviation under Moderate distribution with an example. It is found that the process is in control even when six sigma initiatives are adopted but it is very clear from the comparison that when the process is centered with reduced variation than the 3 sigma control limits, which indicate that the process is not in the level it was expected. So a correction in the process is very much required to reduce the variations. The charts suggested in this research paper will be very useful for the companies practicing six sigma initiatives under Moderate distribution in their process. These charts will replace the existing Shewhart (1931) control charts in future when all the companies started implementing six sigma initiatives under Moderate distribution in their organization.

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