Stress Strength Reliability in Multicomponent Model based on Extended Exponentiated Inverse Lindley Distribution

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Abstract: In this article, the reliability of multicomponent stress strength model has been obtained based on the assumption that stress and strength variate follows independent Exponentiated Extended Inverse Lindley Distribution. For estimating Reliability parameters, maximum likelihood estimation has been used. The reliability estimates are compared asymptotically. Monte Carlo simulation has been performed to study the performance of multicomponent reliability model. Finally real data sets have been used for illustration purpose.

Key words: Stress Strength Reliability, Exponentiated Extended Inverse Lindley, Maximum likelihood Estimator.

I. INTRODUCTION

Reliability is the probability that the system performs adequately without any breakdown for a specific period of time under certain condition. The stress strength model, in the reliability framework, may be defined as the estimation of the system reliability in terms of stress and strength variable. If 'Y' represents the stress on the system and 'X' as strength of the system, then stress strength reliability (SSR), $R = P(X > Y)$ defines the probability that the strength 'X' of the unit transscends the applied arbitrary stress 'Y'. Thus the SSR of a model is the probability that the system acts well till strength exceeds stress. The term SSR was introduced by [14]. In the course of time a lot of research has been done to study the application of model in terms of SSR. The concept of SSR in terms of multicomponent model was first given by [11]. For a complex system having 'k' components, usually iid units, each component having strength $X_1, X_2, \dots X_k$ and each unit is experiencing some arbitrary stress 'Y'. The system will function satisfactorily only if atleast s out of k components will serve the purpose. In this framework , the system has common stress Y and follow distribution function G. The strength of $'s'$ components follow distribution function 'F'. Then the system reliability $R_{s,k}$, as defined by [11] is given as:

$$R_{s,k} = p[atleast\ s\ of\ (X_1, X_2 \dots X_k) exceeds\ Y]$$

$$R_{s,k} = \sum_{i=s}^{k} \binom{k}{i} \int_{-\infty}^{\infty} \left(1 - F(y)\right)^i [F(y)]^{k-i}\, dG(y) \quad (1)$$

Various researchers have estimated the SSR for multicomponent model for different lifetime models. [4] obtained SSR for multicomponent model for log logistic distribution. [1] studied reliability analysis of kumarswamy distribution. [2] consider survival analysis of multicomponent system of generalized exponential distribution with different shape parameters. [3] studied estimation of reliability of s out of k system for Rayleigh distribution.[5] analysed kumaraswamy distribution for its survival analysis. [13] Considered Lindley distribution for estimating multicomponent SSR. Recently [6-9] obtained survival analysis for multicomponent model for various distributions. [10] determined reliability for multicomponent model of Bathtub shape or increasing failure rate function. [12] obtained reliability analysis for power lindley distribution in terms of stress strength reliability model. Further, the SSR for exponentiated inverse power lindley distribution was carried out by [16]. In this article we have considered estimation of system reliability of s out of k components when stress and strength variate follows Extended Exponentiated Inverse Lindley Distribution (EEILD) proposed by [15]..Its cdf and pdf are given respectively as:

$$F(x,\lambda,\sigma,\gamma,\theta) = \left[\left(1+\frac{\gamma\sigma}{(\gamma+\sigma)}\right)e^{-\frac{\sigma}{x^{\lambda}}}\right]^{\theta} \qquad \lambda,\sigma,\gamma,\theta > 0\,; \quad x>0 \qquad (2)$$

$$\mathrm{f}(x,,\sigma,\gamma,\theta) = \frac{\lambda\sigma^2\theta}{\sigma+\gamma}\left(\frac{\gamma+x^{\lambda}}{x^{2\lambda+1}}\right)e^{-\frac{\sigma\theta}{x^{\lambda}}}\left[\left(1+\frac{\gamma\sigma}{(\gamma+\sigma)x^{\lambda}}\right)\right]^{\theta-1} \qquad \lambda,\sigma,\gamma,\theta > 0\,; \quad x>0 \qquad (3)$$

The main purpose of the paper is to estimate reliability of s out of k components when stress and strength both follows EEILD. In section II, the reliability of system model is obtained. The MLE, asymptotic distribution and average length of confidence interval (ACL) of $R_{s,k}$, has been obtained in section III. To measure the performance of reliability estimates Monte-Carlo Simulation has been carried out in section IV. In Section V, the real data sets has been used to illustrate the estimation process. In conclusion some findings are presented in Section VI.

II. SSR FOR MULTICOMPONENT MODEL

Consider two independent random variables X and Y which follows EEILD with parameters $(\lambda,\sigma,\gamma,\theta_1)$ and $(\lambda,\sigma,\gamma,\theta_1)$ respectively . The reliability for EEILD using (2) and (3) is given as

$$R_{s,k} = \sum_{i=s}^{k}\binom{k}{i}\int_0^{\infty}\left(1-\left(\left(1+\frac{\sigma\gamma}{\gamma+\sigma}\frac{1}{x^{\lambda}}\right)e^{-\frac{\sigma}{x^{\lambda}}}\right)^{\theta_1}\right)^{i}\times\left(\left(\left(1+\frac{\gamma\,\sigma}{\gamma+\sigma}\frac{1}{x^{\lambda}}\right)e^{-\frac{\sigma}{x^{\lambda}}}\right)^{\theta_1}\right)^{k-i}\frac{\lambda\,\sigma^2\,\theta_2}{\gamma+\sigma}\times$$
$$\left(\frac{\gamma+x^{\lambda}}{x^{2\lambda+1}}\right)e^{-\frac{\sigma\,\theta_2}{x^{\lambda}}}\left(1+\frac{\gamma\sigma}{\gamma+\sigma}\frac{1}{x^{\lambda}}\right)^{\theta_2-1}\partial x$$

$$R_{s,k} = \theta\sum_{i=s}^{k}\binom{k}{i}\int_0^1(1-t)^i t^{k+\theta-i-1}\,dt$$

If $t=\left(\left(1+\frac{\gamma\sigma}{\gamma+\sigma}\frac{1}{x^{\lambda}}\right)e^{-\frac{\sigma}{x^{\lambda}}}\right)^{\theta_1}$ and $\theta=\frac{\theta_2}{\theta_1}$

$$= \theta\sum_{i=s}^{k}\binom{k}{i}\beta[k+\theta-i,i+1]$$

$$= \theta\sum_{i=s}^{k}\frac{k!}{i!(k-i)!}\frac{\Gamma(k+\theta-i)\Gamma(i+1)}{\Gamma(k+\theta+1)}$$

$$R_{s,k} = \theta\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}(k+\theta-j)\right]^{-1} \qquad (4)$$

III. THE MAXIMUM LIKELIHOOD ESTIMATOR (MLE) OF $R_{s,k}$

Suppose X~ EEILD ($\lambda,\sigma,\gamma,\theta_1$) of size n_1 and Y~EEILD ($\lambda,\sigma,\gamma,\theta_2$) of size n_2 then, the log-likelihood function is given as:

$$l(\lambda,\sigma,\theta_1,\theta_2) = n_1[ln\ \lambda+2ln\,\sigma+ln\ \theta_1-\ln(\gamma+\sigma)]+\sum_{h=1}^{n_1}\ln(\gamma+x_h^{\lambda})-(2\lambda+1)\sum_{h=1}^{n_1}\log x_i -$$
$$\sigma\ \theta_1\sum_{h=1}^{n_1}\frac{1}{x_h^{\lambda}}+(\theta_1-1)\sum_{h=1}^{n}\log\left(1+\frac{\sigma\gamma}{\gamma+\sigma}\frac{1}{x_h^{\lambda}}\right)+n_2[ln\ \lambda+2ln\,\sigma+ln\ \theta_2-\ln(\gamma+\sigma)]+\sum_{j=1}^{n_2}\ln\big(\gamma+$$
$$y_j^{\lambda}\big)-(2\lambda+1)\sum_{j=1}^{n_2}\ln y_j-\sigma\theta_2\sum_{j=1}^{n_2}\frac{1}{y_j^{\lambda}}+(\theta_2-1)\sum_{j=1}^{n_2}\ln\left(1+\frac{\sigma\gamma}{1+\sigma\gamma}\frac{1}{y_j^{\lambda}}\right) \qquad (5)$$

The MLEs $\hat{\lambda},\hat{\sigma},\hat{\gamma},\hat{\theta}_1\hat{\theta}_2$, can be estimated as a solution of

$$\frac{\partial l}{\partial\lambda}=0 \Rightarrow \frac{n_1}{\lambda}+\sum_{h=1}^{n_1}\frac{x_h^{\lambda}ln(x_h)}{\gamma+x_h^{\lambda}}-2\sum_{h=1}^{n_1}lnx_h+\sigma\theta_1\sum_{h=1}^{n_1}x_h^{\lambda}lnx_h-\frac{(\theta_1-1)\gamma\,\sigma}{\gamma+\sigma}\sum_{h=1}^{n_1}\left[\frac{x_h^{\lambda}lnx_h}{\left(1+\frac{\gamma\sigma}{(\gamma+\sigma)x_h^{\lambda}}\right)}\right]$$

$$\frac{n_2}{\lambda}+\sum_{j=1}^{n_2}\frac{y_j^{\lambda}ln(y_j)}{\gamma+y_j^{\lambda}}-2\sum_{j=1}^{n_2}lny_j+\sigma\theta_2\sum_{j=1}^{n_2}y_j^{\lambda}lny_j-\frac{(\theta_2-1)\gamma\sigma}{\gamma+\sigma}\sum_{j=1}^{n_2}\left[\frac{y_j^{\lambda}lny_j}{1+\frac{\gamma\sigma}{(\gamma+\sigma)y_j^{\lambda}}}\right]=0 \qquad (6)$$

$$\frac{\partial l}{\partial \sigma}=0 => \frac{2\ n_1}{\sigma}-\frac{n_1}{1+\sigma}-\quad \theta_1 \sum_{h=1}^{n_1}\frac{1}{x_h^{\lambda}}+(\ \theta_1-1)\sum_{h=1}^{n_1}\left[\frac{1}{1+\frac{\gamma\sigma}{(\gamma+\sigma)x_h^{\lambda}}}\right]\left[\frac{\gamma^2}{(\gamma+\sigma)x_h^{\lambda}}\right]+$$

$$\frac{2\ n_2}{\sigma}-\frac{n_2}{1+\sigma}-\quad \theta_2 \sum_{j=1}^{n_2}\frac{1}{y_j^{\lambda}}+(\theta_2-1)\sum_{h=1}^{n_1}\left[\frac{1}{1+\frac{\gamma\sigma}{(\gamma+\sigma)y_j^{\lambda}}}\right]\left[\frac{\gamma^2}{(\gamma+\sigma)y_j^{\lambda}}\right]=0 \tag{7}$$

$$\frac{\partial l}{\partial \gamma}=0 => \frac{n_1}{\gamma+\sigma}+\sum_{h=1}^{n_1}\frac{1}{\gamma+x_h^{\lambda}}+(\ \theta_1-1)\sum_{h=1}^{n_1}\left[\frac{1}{1+\frac{\gamma\sigma}{(\gamma+\sigma)x_h^{\lambda}}}\right]\left[\frac{\sigma^2}{(\gamma+\sigma)x_h^{\lambda}}\right]+$$

$$\frac{n_2}{\gamma+\sigma}+\sum_{j=1}^{n_2}\frac{1}{\gamma+y_j^{\lambda}}+(\ \theta_2-1)\sum_{j=1}^{n_2}\left[\frac{1}{1+\frac{\gamma\sigma}{(\gamma+\sigma)y_j^{\lambda}}}\right]\left[\frac{\sigma^2}{(\gamma+\sigma)y_j^{\lambda}}\right] \tag{8}$$

$$\frac{\partial l}{\partial\ \theta_1}=0 => \frac{n_1}{\theta_1}-\ \sigma\sum_{h=1}^{n_1}\frac{1}{x_h^{\lambda}}+\sum_{h=1}^{n_1}\ln\left(1+\frac{\gamma\sigma}{\gamma+\sigma}\frac{1}{x_h^{\lambda}}\right)=0 \tag{9}$$

$$\frac{\partial l}{\partial\ \theta_2}=0 => \frac{n_2}{\theta_2}-\ \sigma\sum_{j=1}^{n_2}\frac{1}{y_j^{\lambda}}+\sum_{j=1}^{n_2}\ln\left(1+\frac{\gamma\sigma}{\gamma+\sigma}\frac{1}{y_j^{\lambda}}\right)=0 \tag{10}$$

Solving (9) and (10) we obtain

$$\hat{\theta}_{\mathbf{1}}=\frac{n_1}{\sigma\sum_{h=1}^{n_1}\frac{1}{x_h^{\lambda}}+\sum_{h=1}^{n_1}\ln\left(1+\frac{\gamma\sigma}{\gamma+\sigma}\frac{1}{x_h^{\lambda}}\right)}$$

$$\hat{\theta}_{\mathbf{2}}=\frac{n_2}{\sigma\sum_{j=1}^{n_2}\frac{1}{y_j^{\lambda}}+\sum_{j=1}^{n_2}\ln\left(1+\frac{\gamma\sigma}{\gamma+\sigma}\frac{1}{y_j^{\lambda}}\right)}$$

Using $\hat{\theta}_{\mathbf{1}}$ and $\hat{\theta}_{\mathbf{2}}$ in (4) , we have

$$R_{s,k}=\hat{\theta}\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}\left(k+\hat{\theta}-j\right)\right]^{-1} \qquad where\ \hat{\theta}=\frac{\widehat{\theta}_2}{\widehat{\theta}_1}$$

Now, the Asymptotic Variance (AV) of MLE is obtained as:

$$V(\hat{\theta}_1)=\left[E\left(-\frac{\partial^2 l}{\partial\ \theta_1}\right)\right]^{-1}=\frac{\theta_1^2}{n_1} \quad \text{and}$$

$$V(\hat{\theta}_2)=\left[E\left(-\frac{\partial^2 l}{\partial\ \theta_2}\right)\right]^{-1}=\frac{\theta_2^2}{n_2}$$

The AV of estimate of $R_{s,k}$ as given by Rao [17],

$$AV(\hat{R}_{s,k})=\ V(\hat{\theta}_1)\left(\frac{\partial R_{s,k}}{\partial\ \theta_1}\right)^2+V(\hat{\theta}_2)\left(\frac{\partial R_{s,k}}{\partial\ \theta_2}\right)^2$$

For simplicity of $R_{s,k}$, we find $\hat{R}_{s,k}$ for $(s,k)=(1{,}3)\ and\ (2{,}4)$

$$\frac{\partial \hat{R}_{1,3}}{\partial\ \theta_1}=\frac{3\hat{\theta}}{\hat{\theta}_1(3+\hat{\theta})} \qquad and \qquad \frac{\partial \hat{R}_{1,3}}{\partial\ \theta_2}=\frac{-3}{\hat{\theta}_1(3+\hat{\theta})}$$

$$\frac{\partial \hat{R}_{2,4}}{\partial \Upsilon_1}=\frac{12\hat{\theta}(7+2\hat{\theta})}{\Upsilon_1\left[(3+\hat{\theta})(4+\hat{\theta})\right]^2}$$

$$\frac{\partial \hat{R}_{2,4}}{\partial \Upsilon_2}=\frac{-12(7+\hat{\theta})}{\theta_2\left[(3+\hat{\theta})(4+\hat{\theta})\right]^2}$$

Hence,

$$AV(\hat{R}_{1,3})=\frac{9\hat{\theta}^2}{(3+\hat{\theta})^4}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)$$

$$AV(\hat{R}_{2,4}) = \frac{144\hat{\theta}^2(7+2\hat{\theta})^2}{[(3+\hat{\theta})(4+\hat{\theta})]^4}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)$$

$$As \quad n_1 \to \infty, \; n_2 \to \infty, \qquad \frac{\hat{R}_{s,k}-R_{s,k}}{AV(\hat{R}_{s,k})} \to N(0,1)$$

The asymptotic 95% CI is obtained as $\hat{R}_{s,k} \pm \sqrt{AV(\hat{R}_{s,k})}$

$$\hat{R}_{1,3} \pm 1.96\frac{3\hat{\theta}}{(3+\hat{\theta})^2}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

$$\hat{R}_{2,4} \pm 1.96\frac{12\hat{\theta}(7+2\hat{\theta})}{[(3+\hat{\theta})(4+\hat{\theta})]^2}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

IV. SIMULATION STUDY

To examine the performance of $R_{s,k}$, Monte Carlo simulation study has been performed by considering different sample sizes. A simulation study of 3000 random samples are generated for varying sizes for $(\theta_1,\theta_2) = (2.5,0.5), (2,0.5), (1.5,0.5), (1.0,0.5), (0.5,0.5), (0.5,1.0), (0.5,1.5), (0.5,2.0)$. In the simulation study, we consider λ, σ , γ are known. For each combination of (θ_1,θ_2) for different sample sizes, average bias, average mean square error (MSE), ACL has been obtained. The numerical results have been calculated using R software. The true value of multicomponent SSR with the combination of (θ_1,θ_2) for $(s,k) = (1,3)$ are $(0.9375,\ 0.9230, 0.9, 0.8571, 0.75, 0.6, 0.5, 0.4285), and\ for$ $(2,4) =$ $0.8928, 0.8687, 0.8307, 0.7619, 0.6, 0.4, 0.2857, 0.2142$. It is observed that true value of multicomponent SSR increases as strength parameter θ_1 increases for fixed stress parameter θ_2 and it decreases as θ_2 increases for fixed θ_1 in each case of $(s,k) = (1,3)\ and\ (2,4)$. Further average bias increases as $\theta_2$increases and it decreases as θ_1 as shown in Table 2. Also from Table 3 and 4 it is observed that as sample size increases , the average MSE and ACL decreases, in both the cases of (s,k).

Table 1. *Estimated Reliability* $R_{s,k}$

		(θ_1,θ_2)							
(s,k)	(n,m)	(2.5,0.5)	(2.0,0.5)	(1.5,0.5)	(1.0,0.5)	(0.5,0.5)	(0.5,1.0)	(0.5,1.5)	(0.5,2.0)
(1,3)	(10,10)	0.9678	0.9575	0.9384	0.8951	0.7430	0.4943	0.3562	0.2745
	(15,15)	0.9688	0.9587	0.9401	0.8976	0.7470	0.4988	0.3591	0.2762
	(20,20)	0.9690	0.9590	0.9406	0.8982	0.7486	0.4995	0.3590	0.2757
	(25,25)	0.9692	0.9592	0.9409	0.8987	0.7485	0.4997	0.3586	0.2751
	(30,30)	0.9692	0.9592	0.9408	0.8985	0.7481	0.4989	0.3576	0.2741
	(35,35)	0.9692	0.9592	0.9407	0.8984	0.7477	0.4980	0.3566	0.2730
(2,4)	(10,10)	0.9226	0.9018	0.8672	0.7970	0.5951	0.3346	0.2122	0.1465
	(15,15)	0.9238	0.9036	0.8695	0.7999	0.5985	0.3372	0.2132	0.1466
	(20,20)	0.9242	0.9041	0.8701	0.8008	0.5994	0.3375	0.2128	0.1459
	(25,25)	0.9244	0.9044	0.8704	0.8011	0.5996	0.3372	0.2120	0.1450
	(30,30)	0.9243	0.9043	0.8702	0.8008	0.5991	0.3363	0.21111	0.1440
	(35,35)	0.9242	0.9041	0.8700	0.8005	0.5984	0.3352	0.2099	0.1431

Table 2. Average Bias of Simulated $R_{s,k}$

		(θ_1,θ_2)							
(s,k)	(n,m)	(2.5,0.5)	(2.0,0.5)	(1.5,0.5)	(1.0,0.5)	(0.5,0.5)	(0.5,1.0)	(0.5,1.5)	(0.5,2.0)
(1,3)	(10,10)	0.0303	0.0344	0.0384	0.0379	−0.0060	−0.1056	−0.1437	−0.1540
	(15,15)	0.0313	0.0356	0.0401	0.0404	−0.0029	−0.1011	−0.1408	−0.1522
	(20,20)	0.0315	0.0359	0.0406	0.0411	−0.0019	−0.1004	−0.1409	−0.1527
	(25,25)	0.0317	0.0362	0.0409	0.0415	−0.0014	−0.1002	−0.1413	−0.1534
	(30,30)	0.0317	0.0361	0.0408	0.0414	−0.0018	−0.1010	−0.1423	−0.1544
	(35,35)	0.0317	0.0361	0.0407	0.0412	−0.0022	−0.1019	−0.1433	−0.1554
	(10,10)	0.0294	0.0330	0.0364	0.0351	−0.0048	−0.0653	−0.0734	−0.0677

(2,4)	(15,15)	0.0309	0.0348	0.0387	0.0380	−0.0014	−0.0627	−0.0734	−0.0676
	(20,20)	0.0313	0.0354	0.0393	0.0389	−0.0005	−0.0624	−0.0728	−0.0683
	(25,25)	0.0316	0.0356	0.0397	0.0392	−0.0003	−0.0627	−0.0736	−0.0692
	(30,30)	0.0314	0.0355	0.0394	0.0389	−0.0008	−0.0636	−0.0746	−0.0701
	(35,35)	0.0313	0.0353	0.0392	0.0386	−0.0015	−0.0647	−0.0757	−0.0711

$Table3\ \ Average\ MSE\ of\ Simulate\ estimates\ of\ R_{s,k}$

		(θ_1,θ_2)							
(s,k)	(n,m)	(2.5,0.5)	(2.0,0.5)	(1.5,0.5)	(1.0,0.5)	(0.5,0.5)	(0.5,1.0)	(0.5,1.5)	(0.5,2.0)
(1,3)	(10,10)	0.0011	0.0014	0.0021	0.0029	0.0040	0.0208	0.0292	0.0304
	(15,15)	0.0010	0.0014	0.0020	0.0025	0.0030	0.0167	0.0257	0.0277
	(20,20)	0.0010	0.0014	0.0020	0.0023	0.0022	0.0151	0.0244	0.0268
	(25,25)	0.0010	0.0014	0.0019	0.0022	0.0017	0.0140	0.0235	0.0263
	(30,30)	0.0010	0.0014	0.0019	0.0021	0.0015	0.0135	0.0232	0.0261
	(35,35)	0.0009	0.0013	0.0018	0.0020	0.0013	0.0132	0.0231	0.0261
(2,4)	(10,10)	0.0015	0.0021	0.0030	0.0044	0.0060	0.0124	0.0112	0.0084
	(15,15)	0.0014	0.0018	0.0026	0.0035	0.0040	0.0095	0.0092	0.0071
	(20,20)	0.0013	0.0017	0.0023	0.0030	0.0030	0.0082	0.0084	0.0066
	(25,25)	0.0012	0.0016	0.0022	0.0027	0.0023	0.0073	0.0078	0.0063
	(30,30)	0.0012	0.0015	0.0021	0.0025	0.0020	0.0069	0.0076	0.0062
	(35,35)	0.0011	0.0015	0.0020	0.0023	0.0015	0.0066	0.0074	0.0061

$Table4.\ \ Average\ length\ of\ Confidence\ Interval\ of\ simulate\ estimates\ of\ R_{s,k}$

		(θ_1,θ_2)							
(s,k)	(n,m)	(2.5,0.5)	(2.0,0.5)	(1.5,0.5)	(1.0,0.5)	(0.5,0.5)	(0.5,1.0)	(0.5,1.5)	(0.5,2.0)
(1,3)	(10,10)	0.0383	0.0501	0.0709	0.1147	0.2313	0.2983	0.2740	0.2390
	(15,15)	0.0305	0.0399	0.0566	0.0922	0.1886	0.3469	0.2275	0.1982
	(20,20)	0.0262	0.0343	0.0488	0.0797	0.1637	0.2154	0.1983	0.1725
	(25,25)	0.0234	0.0306	0.0435	0.0712	0.1465	0.1937	0.1782	0.1548
	(30,30)	0.0214	0.0280	0.0398	0.0652	0.1344	0.1773	0.1630	0.1414
	(35,35)	0.0198	0.0260	0.0370	0.0605	0.1248	0.1646	0.1511	0.1309
(2,4)	(10,10)	0.0900	0.1116	0.1462	0.2086	0.3298	0.3361	0.2717	0.2148
	(15,15)	0.0724	0.0899	0.1181	0.1694	0.2704	0.2788	0.2257	0.1780
	(20,20)	0.0625	0.0777	0.1021	0.1467	0.2349	0.2432	0.1967	0.1547
	(25,25)	0.0558	0.0694	0.0913	0.1314	0.2108	0.2186	0.1766	0.1386
	(30,30)	0.0511	0.0635	0.0836	0.1203	0.1929	0.2000	0.1613	0.1264
	(35,35)	0.0474	0.0590	0.0776	0.0117	0.1791	0.1855	0.1493	0.1168

V. INDUSTRIAL APPLICATION

In this section,the methodology with the following two data sets has been illustrated, which were initially given by Bader and Priest [18], on failure stresses (in GPa) of single carbon fibres of lengths $20mm(X)$ and $50mm(Y)$ respectively.

$X =$ 1.312,1.314,1.479,1.552,1.700,1.803, 1.861,1.865, 1.944, 1.958, 1.966, 1.997, 2.006, 2.021, 2.027, 2.055, 2.063, 2.098,2.14, 2.179, 2.224, 2.240, 2.253, 2.270, 2.272, 2.274, 2.301, 2.301,2.301, 2.359, 2.382, 2.382, 2.426, 2.434, 2.435,2.478, 2.490, 2.511,2.514,2.535,2.554, 2.566,2.571, 2.586, 2.629, 2.633, 2.642, 2.648, 2.684, 2.697,2.726, 2.770, 2.773, 2.800, 2.809, 2.818,2.821,2.848,2.88, 2.954, 3.012, 3.067, 3.084, 3.090, 3.096, 3.128,3.233, 3.433, 3.585,3.585.

$Y =$ 1.339,1.434, 1.549, 1.574, 1.589, 1.613, 1.746,1.753,1.764,1.807,1.812,1.84,1.852,1.852,1.862, 1.864,1.931,1.952,1.974,2.019,2.051,2.055, 2.058,2.088,2.125,2.162,2.171,2.172,2.18,2.194,2.211, 2.27,2.272,2.28,2.299,2.308,2.335,2.349,2.356,2.386, 2.39,2.41, 2.43,2.431, 2.458, 2.471,2.497, 2.514,2.558,2.577,2.593,2.601,2.604, 2.62,2.633,2.67,2.682, 2.699,2.705, 2.705, 2.735,2.785, 3.02, 3.042, 3.116, 3.174.

First we assume that $X \sim EEILD(\lambda_1, \sigma_1, \gamma_1, \theta_1)$ and $Y \sim EEIPLD(\lambda_2, \sigma_2, \gamma_2, \theta_2)$.The MLE of unknown parameters are:

$\hat{\lambda}_1 = 4.1268$ $\hat{\sigma}_1 = 5.7683$ $\hat{\gamma}_1 = 0.0025$ $\hat{\theta}_1 = 4.0357$

$\hat{\lambda}_2 = 4.9925$ $\hat{\sigma}_2 = 7.6198$ $\hat{\gamma}_2 = 0.0516$ $\hat{\theta}_2 = 4.1848$

and the corresponding log likelihood value is $-2\ logL_1 = 214.96$

Now, if we suppose that that $X \sim EIPLD(\lambda, \sigma, \gamma, \theta_1)$ and $Y \sim EIPLD(\lambda, \sigma, \gamma, \theta_2)$.The MLE of unknown parameters are:

$\hat{\lambda} = 4.4696$ $\hat{\sigma} = 4.7679$ $\hat{\gamma} = 0.00002$ $\hat{\theta}_1 = 6.0371$ $\hat{\theta}_2 = 4.8506$

and the corresponding log likelihood value is $-2logL_0 = 217.44$

In order to check the equality of parameters , we set up the null and alternate hypothesis as:

$H_0: \lambda_1 = \lambda_2$; $\sigma_1 = \sigma_2$; $\gamma_1 = \gamma_2$ v/s $H_1: \lambda_1 \neq \lambda_2$; $\sigma_1 \neq \sigma_2$; $\gamma_1 \neq \gamma_2$

It is seen that likelihood ratio test statistic is $(-2\ logL_0) - (-2\ logL_1) = 2.48$. The critical value of χ^2 at 5% los for 3 dof is 5.99, Therefore, we accept H_0 and conclude that our assumption $\lambda_1 = \lambda_2$; $\sigma_1 = \sigma_2$; $\gamma_1 = \gamma_2$ is reasonably justified.

Further, the efficacy of the model has been examined. The histogram and Q-Q plots for fitted EEILD model for data sets has been plotted which is displayed in Fig 1, 2, 3 and 4 respectively. Also, to test the goodness of fit, Kolmogorov-Smirnov (K.S) test statistics has been used for both data sets separately. It is veiwed that the K-S distance for data set of length 20mm is 0.15574 with p.value 0.07037 and the k-s distance for data set of length 50mm is 0.15024 with p. value 0.1063. Therefore the 2 data sets are practically fit for EEILD. Using estimates of $\hat{\theta}_1$ and $\hat{\theta}_2$, the MLE of $R_{s,k}$ is

$\hat{R}_{1,3} = 0.7786$ with the 95% CI $(0.5906, 0.7413)$ and

$\hat{R}_{2,4} = 0.6417$ with the 95% CI $(0.5553, 0.7280)$

Fig. 1: Histogram and theoritical densities

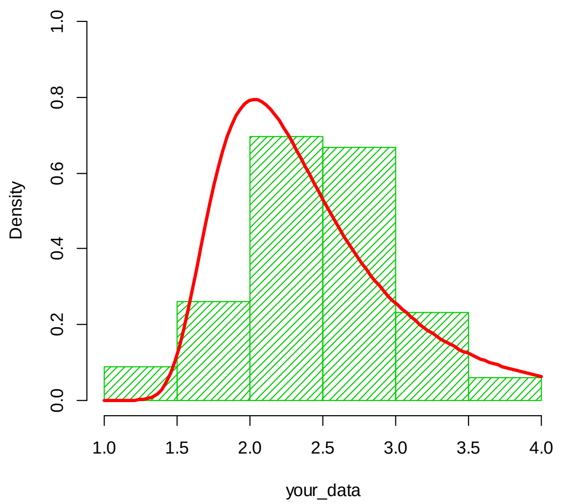


Fig. 2: Histogram and theoritical densities

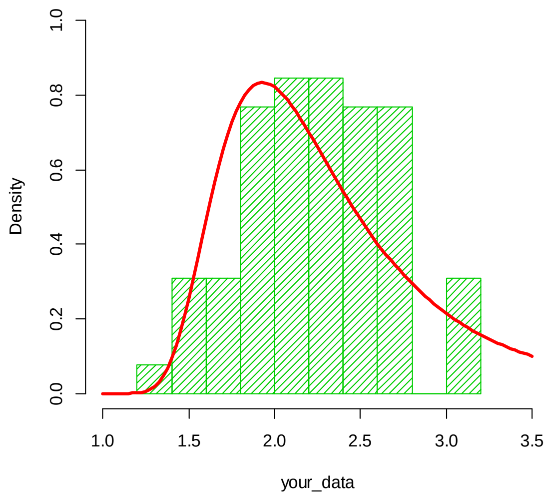


Fig3: QQ plot for EEILD for length 20 mm data

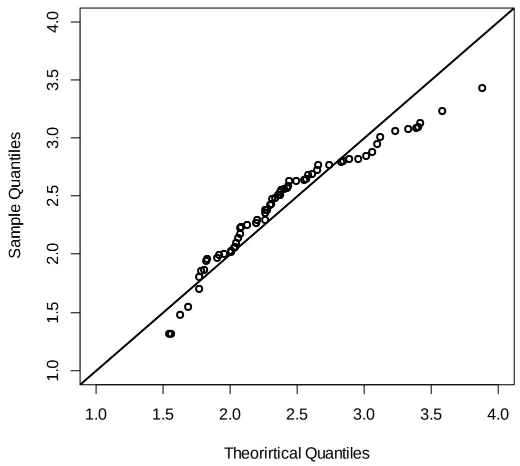
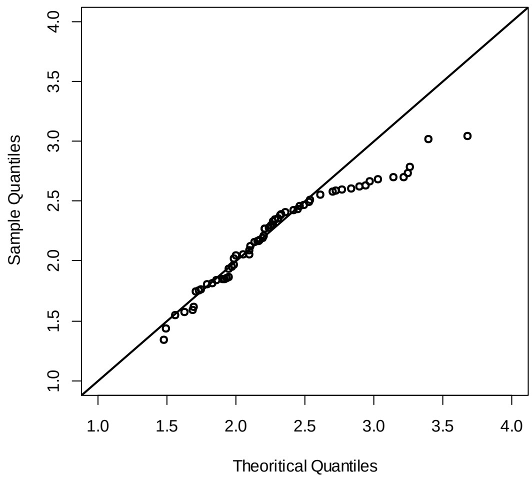


Fig. 4: QQ plot for EEILD for length 50 mm data



VI. CONCLUSION

In this paper, the problem for assessing survival in multicomponent stress strength model has been considered when both stress and strength variate follows EEILD. The reliability of system model has been estimated by using maximum likelihood approach. The simulation outcomes indicates that the reliability of the multicomponent stress strength model increases as θ_1 rises and SSR for multicomponent model decreases as θ_2 increases. Also the average MSE and ACL decreases with increase in sample size. Further the average bias decreases as θ_1 increases and it increases as θ_2 increases. Furthermore, the real data sets has been used to validate the accessibility of the model. The histogram and QQ plot for the fitted EEILD has been plotted for the data sets which is shown in Fig. 1,2,3 and 4 respectively and concluded that the given data sets are relatively fit for EEILD.

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