ELECTRON DYNAMICS IN ACOUSTICALLY DRIVEN SEMICONDUCTOR SUPERLATTICES

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ABSTRACT
Exploring single electron dynamics in semiconductor superlattices driven by an acoustic wave, the types of electron trajectories obtain depend on the strength of the wave amplitude. Two dynamical regimes that are obtained depend on the wave amplitude strength and the initial position of electrons in the acoustic wave. The frequency range of the oscillation produced can be as large as terahertz.

1. Introduction

A Semiconductor Superlattice is a device of periodic layers, a periodic crystal lattice. Two or more semiconductors of similar lattice constants but different band gaps are usually deposited alternatively on each other to form Superlattice. The width of the superlattices is of the orders of magnitude larger than the lattice constant, and is limited by the growth of the structure [1]. This is an example of man-made crystalline structure in which the lattice period is much longer than that of natural crystal that made the unique layered structure to be able to suppress inter miniband tunnelling which thus enhance the electron injection [2].
The Semiconductor Superlattice can be fabricated by epitaxial growth technique out of which molecular beam epitaxy (MBE) is well suited for it. Potential barriers confine the electrons in the SLs and this formed the interfaces between different materials. SLs miniband demonstrates a wide range of nonlinear phenomena that have very useful applications in the ultrafast electronics. Single electron transport in a single miniband can triple measured current at room temperature and with a tilted magnetic field it generates a strange type of chaos, which abruptly delocalises electron orbits [3]. In the presence of external electric and magnetic fields, the transport of electrons in miniband does have a complex behavior which results in a number of interesting effects like THz Bloch oscillations [4, 5], dynamical electron confinement, negative differential drift velocity [6], cyclotron-Bloch resonances and dynamical chaos [7]. The difference in the width of the energy gap in different semiconductors forms the boundary of the conductivity band for perfect SLs, which is modulated periodically and leads to the formation of energy miniband [8, 9]. The doping and controlled lattice strain in the SL structure can be combined to achieve a maximum tunable state. SLs can be categorized as weakly or strongly coupled based on the thickness of the barriers separating the quantum wells [7]. The weakly coupled have relatively thick barriers between the quantum wells, which makes the decay length of the electron wave function small and resonant, like the emission of acoustic phonons that occurs in the vertical hopping transport regime [10].

2. Semi-classical model of acoustic wave

A longitudinal coherent acoustic wave propagating along the x-axis of the SL will generate a deformation potential, resulting in the periodic variation of the conduction band edge of the SL. In this analysis the strain wave is taken to travel along the principal growth axis of the SL, therefore the piezo-electric coupling is zero. This means that any piezo-electric potential effects will be negligible and only the mechanism of electron-phonon interaction exists and is equal to the deformation potential [11].
The potential energy obtain as a result of strain of the lattice, S in the semiclassical model will be

$$V_s = DS \qquad (1)$$

The electron-phonon coupling constant is described by D and can be measured experimentally. In previous work it was found to be $\sim 10\, eV$ [11, 12]. The strain, $S(x,t)$ that will be obtained due to the coherent acoustic wave propagating along the x-axis of SLs will be

$$S = -S_0 \sin(k_s x + \omega_S t) \qquad (2)$$

The maximum strain the wave generates is $S_0 < 0.5\%$; k_s is the wave number of the acoustic wave and takes the value within the first half of the SL minizone. If we assume a linear dispersion relation for the frequency of this acoustic wave, $\omega_S = v_S k_s$ and the speed of sound is $v_S = 5000\ ms^{-1}$ from the experiment [10]. Therefore, the maximum strain is

$$S_0 = k_s A \quad (3)$$

A describes the physical displacement of the lattice obtained from the acoustic wave and is called the mechanical displacement amplitude. Therefore, substituting *equations* (2) and (3) into *equation* (1), we obtain the potential energy generated by the acoustic wave as

$$V_S(x,t) = -U \sin(k_s x - \omega_S t) \quad (4)$$

and $U = DS_0$, the wave amplitude.

3. Model of electron dynamics

To describe the single-electron dynamics in the SL that can be driven by an acoustic wave, we formulated model from the semi classical Hamiltonian of the system. The semi-classical Hamiltonian of the system is the sum of the kinetic and potential energy and is described as

$$\hat{H}(x, p_x, t) = E(p_x) + V_s(x,t) \quad (5)$$

The total kinetic energy is the dispersion relation that has been derived as

$$E(p_x) = \frac{\Delta_{SL}}{2}\left[1 - cos\left(\frac{p_x d}{\hbar}\right)\right] \quad (6)$$

and the potential energy the acoustic wave is described as

$$V_s(x,t) = -U \sin(k_s x - \omega_S t) \quad (7)$$

where U is the wave amplitude. We described $U = S_0 D$ as the wave amplitude of the acoustic wave which creates the maximum strain, S_0<0.5% on the deformation potential, D. The wave number, k_s of acoustic waves lies within the inner half of the minizone so the linear dispersion for frequency of the acoustic wave will be $\omega_s = v_s k_s$ where v_s is the speed of the sound wave. Therefore, the Hamiltonian of the system will be

$$\therefore \quad \hat{H} = \frac{\Delta}{2}\left[1 - \cos\frac{p_x d}{\hbar}\right] - U \sin(k_s x - \omega_S t) \quad (8)$$

The semi-classical Hamilton's equations of motion can be obtained from the Hamiltonian in the above equation as

$$v_x = \frac{dx}{dt} = \frac{\partial H}{\partial p_x} = \frac{\Delta d}{2\hbar} \sin\frac{p_x d}{\hbar} \quad (9)$$

$$\frac{dp_x}{dt} = -\frac{\partial H}{\partial x} = k_s U \cos(k_s x - \omega_S t) \quad (10)$$

Equations (9) and (10) can be solved numerically using a 4th order Runge-Kutta algorithm [13-14]. Taking initial position and momentum to be $x = p_x = 0$ when $t = 0$, for time independent, the electron trajectories are determined in the absence of scattering. The electron trajectories for different wave amplitudes can be obtained and analyses of the trajectories help to understand the electron transport in the SL.

4. Discussion and Results

When the acoustic wave is applied along the axis of a SL, the electron dynamics depends on the value of the wave amplitude. The wave amplitude will produce a force on the electrons that causes them to experience acceleration and movement, since energy states are available in the bands.

We illustrate the electron trajectories for different values of wave amplitude. Fig. 1.0 shows the electron trajectory when $U = 1meV$, which was obtained numerically from *equations* (9) and (10). The $x(t)$ trajectory consists of regular oscillations that are almost sinusoidal, and the electron is being dragged in the SL with the velocity of the acoustic wave.

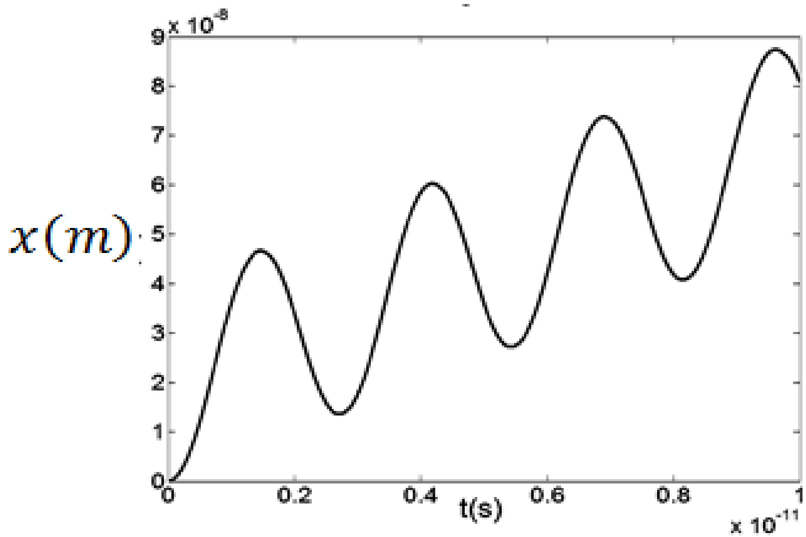

Figure 1.0: The electron trajectory in real space when the wave amplitude, $\boldsymbol{U = 1meV}$, initially at $\boldsymbol{x = 0}$ $\boldsymbol{and}$ $\boldsymbol{p = 0}$, the electron is being dragged in the SL with the velocity of the acoustic wave.

In this region, when $U = 1meV$, the electrons are trapped. The electron will be in the parabolic region of the $E'\,(p_x)$ curve, where the momentum, $p_x = 0$ as shown in Fig. 2.0. The electron is confined within a single potential well in the acoustic wave and oscillates back and forth across the well. $E'\,(p_x)$ is the modified dispersion relation. When U reaches the critical value, U_c this is equal to the local maximum of $E'\,(p_x)$, arrowed in Fig 2.0, the electron is no longer trapped within the acoustic wave but traverse since the wave amplitude, $U > 4meV$.

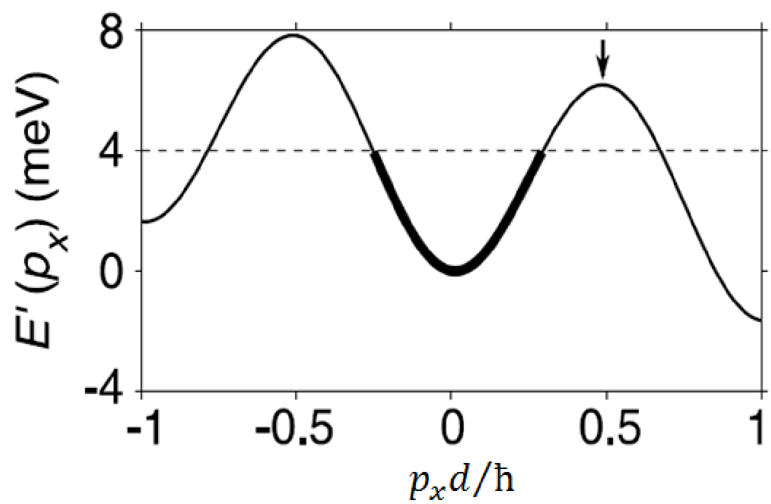

Figure 2.0: The modified dispersion relation is plotted against the momentum showing electron transport in the potential well. The wave amplitude must be greater than the miniband for the electron to traverse over the barrier. At the arrowed peak $\boldsymbol{E'\,(p_x) = U_c}$ refer to as the critical value. The figure is adapted from reference [3].

As the wave amplitude increases, consequently increasing the drift velocity of the system, the electrons begin to Bloch oscillate. Fig. 3.0 is the electron trajectory when the wave amplitude is increasing; the wave amplitude is $7meV$. At this region, the electron has started Bloch oscillate which causes the electron transport to be suppressed. The electron $x(t)$ trajectory consists of Bloch-like oscillations caused by the acoustic wave.

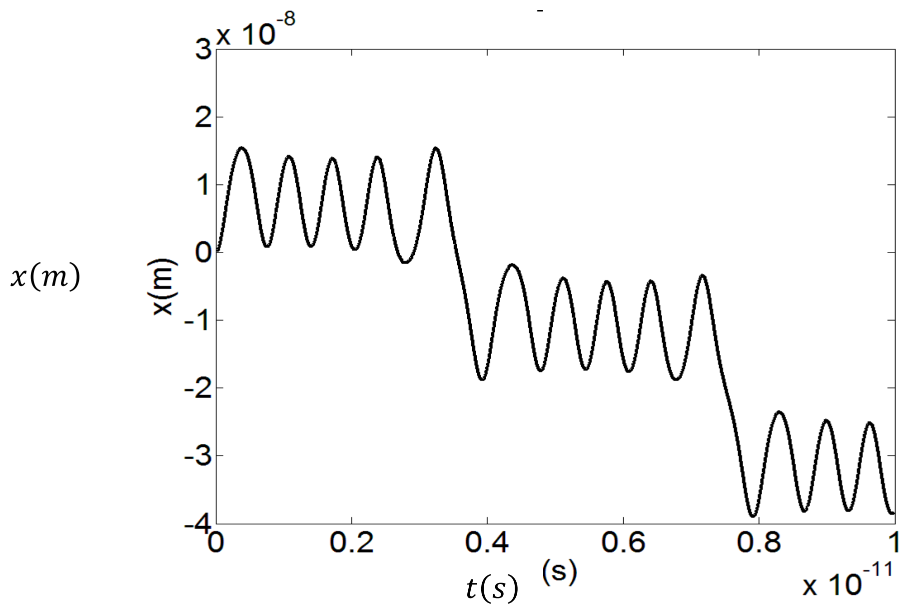

Figure 3.0: Electron trajectory when the wave amplitude, $\boldsymbol{U = 7meV}$. The frequency oscillations are interrupted by jumps in the negative $\boldsymbol{x - direction}$.

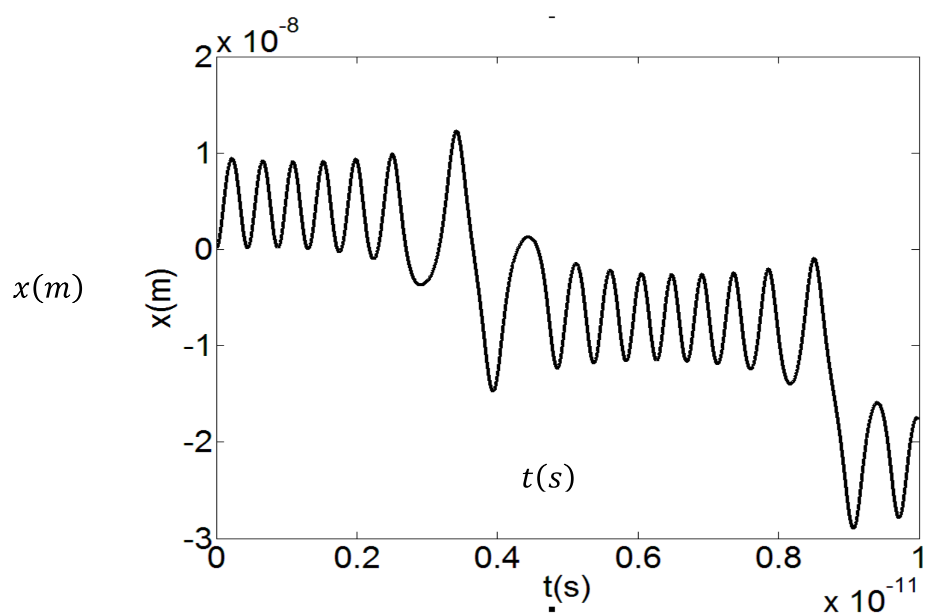

Figure 4.0: Electron trajectory in real space when the wave amplitude is $\boldsymbol{U = 10meV}$ with initial condition $\boldsymbol{x = 0}$ and $\boldsymbol{p = 0}$. The high frequency oscillations are driven by the acoustic wave.

When the wave amplitude is increased further, we have bursts of high-frequency fluctuations in electron trajectory $x(t)$ but they are moving in the negative x −direction. The bursts of Bloch-like oscillations are interrupted by jumps in the negative direction as shown in Fig. 4.0. We could observe a change in trajectory at different strengths of wave amplitude but more investigations are still required.

5. Conclusion

When an external field is applied along the principal axis of SLs, in this case acoustic wave. The dynamics of electron in the SL depend on the value of wave amplitude. There is a qualitative change in the dynamics as we increase the wave amplitudes. At high wave amplitude terahertz (THZ) frequency Bloch oscillations is observed.

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